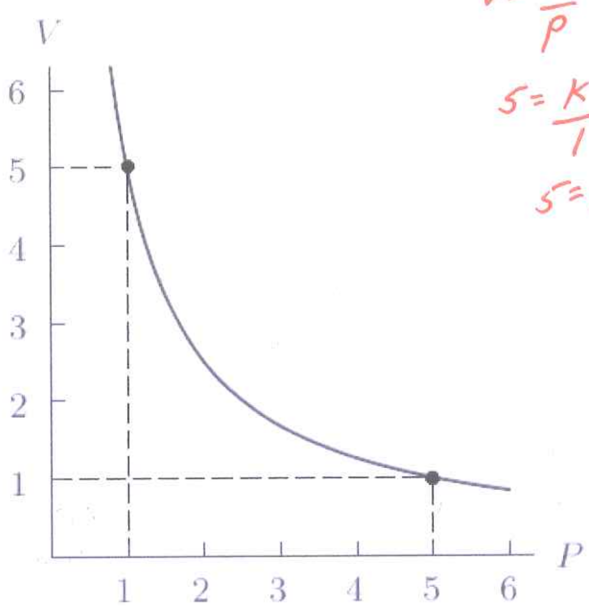
Chapter 11 Review Name: ____________________________

All answers must be justified with work. No work, no credit!

1. When temperature is held constant, the pressure P and volume V of a quantity of gas are inversely proportional (Boyle's Law). The following figure shows this relationship for a particular gas. Find a formula for V in terms of P and use it to find V when P is 12. Round to 2 decimal places.



$V = \frac{K}{P}$

$5 = \frac{K}{1}$

$5 = K$

$V = \frac{5}{P}$

$V = \frac{5}{12}$

$\boxed{V = .42L}$

2. The volume occupied by a fixed quantity of gas such as oxygen is inversely proportional to its pressure, provided that its temperature is held constant. Suppose that a quantity of oxygen occupies a 100 liter volume at a pressure of 16 atmospheres. Write an equation for this inverse variation. If the temperature of the oxygen does not change, how many liters will it occupy if its pressure rises to 20 atmospheres? Round to 1 decimal place.

$V = \frac{K}{P}$

$100 = \frac{K}{16}$

$1600 = K$

$V = \frac{1600}{P}$

$V = \frac{1600}{20}$

$V = 80\text{ L}$

3. Is $y = (x-1)(x+1)$ a power function?

$y = x^2 - 1$ NO

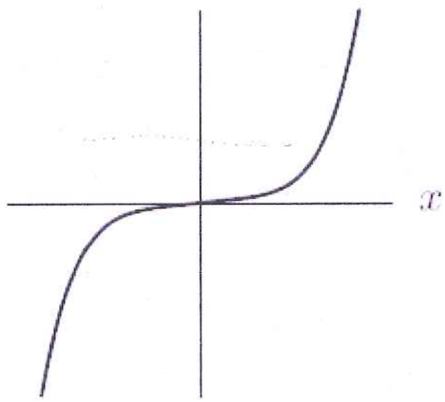
4. The power function through the point (3, 7) and (6, 15) is $y = kx^p$, where $k =$ $\underline{2.092}$ and $p =$ $\underline{1.100}$. Round your answers to 3 decimal places.

$$\frac{y = kx^p}{y = kx^p} \Rightarrow \frac{15 = k6^p}{7 = k3^p} \quad \frac{15}{7} = \left(\frac{6}{3}\right)^p \quad \frac{15}{7} = 2^p$$

$$\log\left(\tfrac{15}{7}\right) = \log(2^p) \quad \frac{\log\left(\frac{15}{7}\right)}{\log 2} = \frac{p \log 2}{\log 2} \quad 1.100 = p$$

$$7 = k(3)^{1.0995} \quad \frac{7}{3^{1.0995}} = k \quad 2.092 = k$$

5. The polynomial f graphed below has leading term ax^n (i.e. $f(x) = ax^n +$ terms of lower degree). We know that a is <u>positive</u> (positive / negative), n is <u>odd</u> (even / odd), and the smallest possible value of n is <u>3</u>.



6. Compute the following limits:

a) $\lim_{x\to\infty}\left(-5x^4 + 7x^3 - 116x^2\right)$ $= \lim_{x\to\infty} -5x^4 = -\infty$

b) $\lim_{x\to-\infty}\left(-5x^4 + 7x^3 - 116x^2\right)$ $\lim_{x\to\infty} -5x^4 = -\infty$

7. Factor and list the zeros of the function $y = x^3 - 2x^2 - 24x$ in ascending order, separated by commas.

$y = x(x^2 - 2x - 24)$

$0 = x(x-6)(x+4)$

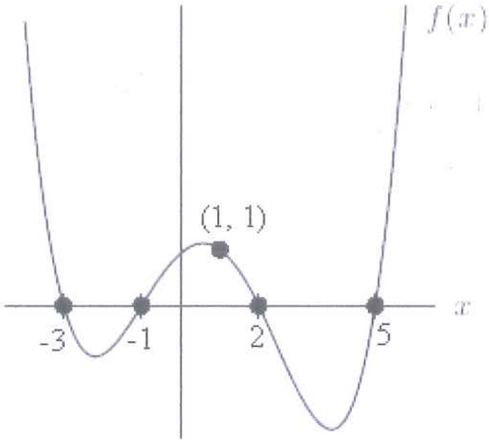
$\boxed{x=0}$ $\quad x - 6 = 0 \quad x + 4 = 0$

$\boxed{x = 6} \quad \boxed{x = -4}$

$-4, 0, 6$

8. Let $f(x) = \dfrac{4}{x+6}$. As $x \to -\infty$, $f(x) \to$ $\underline{0}$.

9. The formula for the function graphed below has leading term ax^n (i.e. $f(x) = ax^n +$ terms of lower degree). If n is as small as possible, then $n =$ __4__ and $a =$ __.03125__.



$f(x) = a(x+3)(x+1)(x-2)(x-5)$

$1 = a(1+3)(1+1)(1-2)(1-5)$

$1 = a(4)(2)(-1)(-4)$

$\frac{1}{32} = \frac{32a}{32}$

$\boxed{.03125 = a}$

10. Which of the following are possible formulas for a fourth degree polynomial with at least one zero at $x = -3$, a double zero at $x = 3$, and long-run behavior: as $x \to \infty$, $y \to -\infty$. .

A) $y = -3(x+3)(x-5)(x-3)^2$ 5^{th} degree

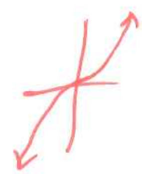
B) $y = 3(x+3)(x-5)(x-3)^2$

C) $y = -(x+3)^2(x-3)^2$

D) $y = -a(x+3)(x-5)(x-3)^2$ where $a > 0$

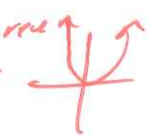
$y = a(x+3)(x-3)^2$

Odd degree if a is +



For $y \to -\infty$ as $x \to \infty$, then a must be negative.

Even degree if a is +



11. Let $f(x) = \dfrac{2+x}{6-8x}$. Find $f^{-1}(x)$.

$y = \dfrac{2+x}{6-8x}$

$\dfrac{x}{1} = \dfrac{2+y}{6-8y}$

$x(6-8y) = 2+y$

$6x - 8xy = 2 + y$

$6x - 2 = y + 8xy$

$\dfrac{6x-2}{1+8x} = \dfrac{y(1+8x)}{1+8x}$

$y = \dfrac{6x-2}{1+8x}$

$\boxed{f^{-1}(x) = \dfrac{6x-2}{8x+1}}$

12. If the function $f(x) = \dfrac{1}{x+4} - \dfrac{x}{x-5}$ is written in the form $f(x) = \dfrac{p(x)}{q(x)}$, a ratio of polynomials, which of the following could be $p(x)$?

A) $-x^2 - 3x - 5$

B) $-x^2 - 4x + 1$

C) $1 - x$

D) -9

$\dfrac{(x-5)}{(x-5)}\dfrac{1}{x+4} - \dfrac{x}{x-5}\dfrac{(x+4)}{(x+4)}$

$\dfrac{x-5}{(x-5)(x+4)} + \dfrac{-x^2-4x}{(x-5)(x+4)} = \dfrac{-x^2-3x-5}{(x-5)(x+4)}$

13. Which of the following are rational functions:

A) $y = \frac{x^2 - 2}{x^5} - \frac{1}{3x^2}$

B) $y = \frac{2^x - 5}{3^x}$ EXPONENTIAL

C) $y = \frac{2\sqrt{x} + 5}{x^3 - 3}$ FRACTIONAL EXPONENT

D) $y = \frac{2x^{-5}}{3 + x^3} + \frac{x^{-2}}{1 - x^{-3}}$

14. Find the long-run behavior of the function $y = \frac{x^2 - 4}{x^5} - \frac{1}{5x^2}$.

$\frac{x^2}{x^5} \quad \frac{1}{x^2} \rightarrow \frac{1}{x^3} + \frac{1}{x^2}$

as $x \to \infty$ $y \to 0$

as $x \to -\infty$ $y \to 0$

15. A 15 kg sample of a certain alloy (mixture of metals) contains 2 kg of tin and 13 kg of copper. A chemist decides to study the properties of the alloy as its percentage of tin is varied. Suppose x represents the quantity of tin, in kg, the chemist adds to the sample. Let $f(x)$ represent the fraction of the mixture's mass composed of tin--that is, the ratio of the tin's mass to the mixture's total mass. A negative value of x represents a quantity of tin removed from the original 15 kg sample. $f^{-1}(0.7) =$ 28.33 kg. Round to 2 decimal places.

$f(x) = \frac{2 + x}{15 + x}$

$.7 = \frac{2 + x}{15 + x}$

$.7(15 + x) = 2 + x$

$10.5 + .7x = 2 + x$

$8.5 = .3x$

$28.33 = x$

16. Determine the vertical and horizontal asymptotes, if they exist, of the function

$y = 3 - \frac{11}{2x + 12} + \frac{1}{7x^4}$

VA: $2x + 12 = 0$ $\quad 7x^4 = 0$

$2x = -12$ $\quad x^4 = 0$

$\boxed{x = -6 \quad x = 0}$

HA: $3 - 0 + 0$

$\boxed{y = 3}$

17. Which of the following statements are true for $f(x) = \frac{x^2 - 25}{x^2 + 6x}$? Mark all that apply.

A) The x- intercepts are 0 and –6.
B) The x- intercepts are –5 and 5.
C) There are no x-intercepts.
D) The y- intercepts are 0 and –6.
E) The y- intercepts are –5 and 5.
F) There are no y-intercepts.

$\frac{(x+5)(x-5)}{x(x+6)}$

x-int: $x + 5 = 0$ $\quad x - 5 = 0$

$x = -5$ $\quad x = 5$

y-int: $f(0) = \frac{0^2 - 25}{0^2 + 6(0)} = \frac{-25}{0}$

undefined – no y-int

18. Which of the following statements describe the graph of $f(x)=\dfrac{x+3}{x+1}$? Mark all that apply.

A) There is a vertical asymptote at x = -1.
B) There is a vertical asymptote at x = 1.
C) There is a horizontal asymptote at y = -3.
D) There is a horizontal asymptote at y = 1.
E) There is an x-intercept at x = -3.
F) There is an x-intercept at x = 0.
G) There is a y-intercept at y = 3.
H) There is a y-intercept at y = 0.

x-int: $x+3=0$
$x=-3$

y-int
$f(0)=\dfrac{0+3}{0+1}=3$
$(0,3)$

VA: $x+1=0$
$x=-1$

HA: $y=\dfrac{x}{x}$
$y=1$

19. Estimate the following limits

$\boxed{-\infty}$ A) $\lim\limits_{x\to3+}\dfrac{6x^2}{(x-3)^3(x-5)}$

as $x\to3+$ $6x^2\to54$ $(x-3)^3\to+0$ $(x-5)\to-2$
$(x-3)^3(x-5)\to{}^-0$ (very small negative #)
$\dfrac{54}{-0}=-\infty$

$\boxed{0}$ B) $\lim\limits_{x\to3+}\dfrac{(x-3)^3(x-5)}{6x^2}$

$\dfrac{-0}{54}=0$

20. Which of the following functions dominates as $x\to\infty$?

A) $y=(10x)^{1.4}$
B) $y=1.4^x$ Exponential always ends up larger than any power function

21. As $x\to\infty$, $f(x)=\dfrac{3^x-5}{4^x+5}\to$ __0__. Enter "infinity " or "- infinity" for ∞ or $-\infty$.

will dominate 3^x

22. The formula for the exponential function shown in the following table is $g(x)=ab^x$, where a = __4.19985__ and b = __1.40002__. Round both answers to 5 decimal places. Show your work. (Do not use regression function on calculator.)

x	2	3	4	5
$f(x)$	8.232	11.525	16.135	22.589

$y=ab^x$
$y=ab^x$
$11.525=ab^3$
$8.232=ab^2$
$\dfrac{11.525}{8.232}=b^1$
$1.40002=b$

$y=ab^x$
$8.232=a(1.4000243)^2$
$\dfrac{8.232}{1.4000243^2}=a$
$4.19985=a$

23. One of the following tables of data comes from a linear function, one from an exponential function, and one from a power function. The formula for the power function is $y = ax^b$ with $a =$ <u>-2</u> and $b =$ <u>-3</u>. (Show your work!)

x	-2	-1	0	1	2
$f(x)$	6.4	0.4	0	0.4	6.4

EXPONENTIAL

x	5	10	15	20	25
$g(x)$	2.93	4.43	5.93	7.43	8.93

LINEAR

x	-2	-1	0	1	2
$h(x)$	0.25	2	undefined	-2	-0.25

$y = ax^b$

$\dfrac{y = ax^b}{y = ax^b}$

$\dfrac{2 = \cancel{a}(-1)^b}{.25 = \cancel{a}(-2)^b}$

$8 = \left(\frac{1}{2}\right)^b$

$\log 8 = \log\left(\frac{1}{2}\right)^b$

$\dfrac{\log 8}{\log(\frac{1}{2})} = \dfrac{b \log(\frac{1}{2})}{\log(\frac{1}{2})}$

$-3 = b$

$y = ax^b$

$2 = a(-1)^{-3}$

$2 = a\dfrac{1}{(-1)^3}$

$2 = -1a$

$-2 = a$

$y = -2x^{-3}$

24. George measures the force exerted by a certain spring when it is stretched various distances beyond its natural length. The data he gathers is compiled in the following table. He wishes to describe the force as a function of the stretched distance, and decides to use a quadratic function model. After fitting a quadratic function to the data using a calculator, he uses it to estimate the force when the spring is stretched 22 inches. What is his function and approximation (to three decimal places) of this force?

Stretched Distance (feet)	0.17	0.56	0.73	0.81	1.13
Force (pounds)	16.74	499.82	1180.23	1645.11	4102.94

$y = 5364.993x^2 - 2722.895x + 327.128$

$y = 5364.993\left(\frac{22}{12}\right)^2 - 2722.895\left(\frac{22}{12}\right) + 327.128$

$\boxed{y = 13,367.490}$

Can be found on table or graph